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Rational Expectations with Market Power -  
The Paradox of the Disadvantageous Tariff on Oil

by

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## Introduction

The theoretical analysis of exhaustible resources has to date largely ignored the "geo-political realities" which preoccupy policy makers and has concentrated instead on the logically prior problem of analyzing market equilibrium in an autarkic economy. Autarky is a useful framework for the study of competitive equilibrium but is ill-suited to analyze the taxation of those exhaustible resources which are traded between large sovereign nation-states.

One feature that internationally traded commodities such as oil share is the difficulty countries have enforcing futures contracts for these commodities. This property, together with the irreversibility of time - the fact that the past cannot be replayed - is crucial to our analysis and leads to the rather counter-intuitive conclusion that a large importing country may actually be harmed by its market power. Even when enforceable futures contracts are possible, moreover, the importer may have trouble availing himself of them in the absence of binding agreement not to open future spot markets.

An assumption essential for obtaining these results is that a nation's tariff jurisdiction be limited, that it cannot, in general, drive a tax wedge between all consumers and producers. Because tariffs on exhaustible resources figure so prominently in our discussion, we shall study them in some detail. Many economists, if asked to identify the important difference between domestic and trade taxation, would argue that trade taxes are necessarily distortionary, whilst domestic taxes on rents, pure profits, etc. need not be. The first task of this paper is to demonstrate that this distinction is often irrelevant for exhaustible resources. In section 1, we shall establish that, under certain assumptions, an ad valorem tax on output is equivalent to a rent tax and, therefore, non-distortionary.

In a world of only one oil importing country (and many oil producers), the importer effectively has complete tax jurisdiction and can tax the producers' rent efficiently with an import tariff. Indeed, with constant extraction costs the importer can extract (almost) all the rent. If, however, other countries import oil, our importer may lose his tax power with a vengeance, even if (in fact, especially if) he remains a large importer, while others constitute a more competitive fringe. As we suggested above, his problem arises because he cannot enter, explicitly or implicitly, into long-term contracts with foreign suppliers, or more accurately cannot be bound to such contracts. Suppliers' current actions depend on the future behaviour of our importer, but, in the absence of binding contracts, they may have difficulty forecasting this behaviour.

In section 2, we exhibit a model where this difficulty occurs. In section 3, we introduce a much simpler model, which we adopt for its analytic convenience. We also discuss myopic expectations on the part of suppliers as a way of "predicting" future actions of the importer. In our simple model  
In section 4, we consider rational expectations/and show that, in this case, our importer may be severely disadvantaged. After providing a numerical example (in section 5) which demonstrates that the importer may be left worse off than a pure competitor in his position, we investigate what happens when he can impose quotas (i.e. when he can ration suppliers). Up to this point, we suppose that importing countries find oil too costly to store. In sections 7 and 8, we drop this assumption and permit costless storage. We discuss our main results in section 9. In section 10, we investigate conditions under which behavior in our model coincides with that which would occur under binding contracts. Finally, section 11 mentions some of the connections of our analysis with other work.

It is a familiar proposition of conventional tax theory that a tax on rent (or pure profits, correctly defined) is non-distortionary. This is also true for exhaustible resources, using the natural definition of rent. if the future is predictable.

Suppose the cost of extracting  $x$  units of exhaustible resources (called oil, for brevity) at date  $t$ , when the remaining stock is  $S$  is  $C(x, S, t)$ . Suppose also that there is a perfect substitute which can be produced indefinitely from a backstop technology at a cost  $B(z, t)$  for supply  $z$ .<sup>1</sup> Let the dollar value of consumption be  $U(x + z, t)$  so that the efficient path solves

$$(1) \quad \text{Max} \int_0^{\infty} \left[ U(x + z, t) - C(x, S, t) - B(z, t) \right] e^{-rt} dt$$

subject to  $\dot{x} = -S,$

$$\int_0^{\infty} x dt \leq S_0$$

where  $r$  is the rate of discount and  $S_0$  is the initial stock of oil. The necessary conditions for optimality can be found by applying the Maximum Principle to the Hamiltonian

$$(2) \quad H e^{rt} = U - C - B - \mu x$$

where  $\mu e^{-rt}$  is adjoint to  $S$ . Choosing  $x, z$ , to maximize  $H$  gives

$$(3) \quad \left. \begin{array}{l} \frac{\partial U}{\partial x} - \frac{\partial C}{\partial x} - \mu \leq 0 \\ x \geq 0 \end{array} \right\} \text{complementarily} \quad \left. \begin{array}{l} \frac{\partial U}{\partial z} - \frac{\partial B}{\partial z} \leq 0 \\ z \geq 0 \end{array} \right\} \text{complementarily}$$

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1. This appears the simplest method of handling the post-exhaustion world. It is necessary neither that the substitute be perfect, nor even that it ever be introduced, though in such cases it becomes more difficult to guarantee the existence of an optimum extraction policy. See Dasgupta and Heal (1974).

The shadow price of oil,  $\mu$ , must satisfy

$$\frac{d\mu}{dt} - r\mu = - \frac{\partial H e^{rt}}{\partial S} = \frac{\partial C}{\partial S}$$

Given reasonable assumptions on the functional forms, these equations will have a unique solution of the following form

$$(4) \quad \frac{dp}{dt} = \frac{dc}{dt} + \frac{\partial C}{\partial S} + r(p - c) \quad 0 \leq t \leq T$$

$$p = \frac{\partial B}{\partial z} \quad T \leq t$$

where  $p$  is the demand price for energy,  $U_x$ , or  $U_z$ , and  $c$  is the marginal extraction cost,  $C_x$ . The date of exhaustion,  $T$ , is found as the first date at which the price of oil,  $p$ , has risen to the marginal cost of replacing oil by the backstop. The initial price of oil is low enough to exactly exhaust  $S_0$  by date  $T$ . (We shall assume that it is not too costly to preclude complete exhaustion.) Figure 1 shows a possible configuration where  $z^*(t)$  is the solution to

$$B_z(z, t) = U_z(z, t)$$

and  $T'$  is the date of first introduction of the backstop. If marginal extraction costs are non-decreasing it will be possible to decentralise this optimum plan if resource owners are perfectly competitive and perfectly well informed about future prices. This can be seen as the special case of a zero rent tax in the following. Let rents, defined as  $px - C$  be taxed at a constant rate  $\tau$ , so that producers choose  $x$  to maximise

$$(5) \quad \int_0^{\infty} (1 - \tau) (px - C) e^{-rt} dt$$

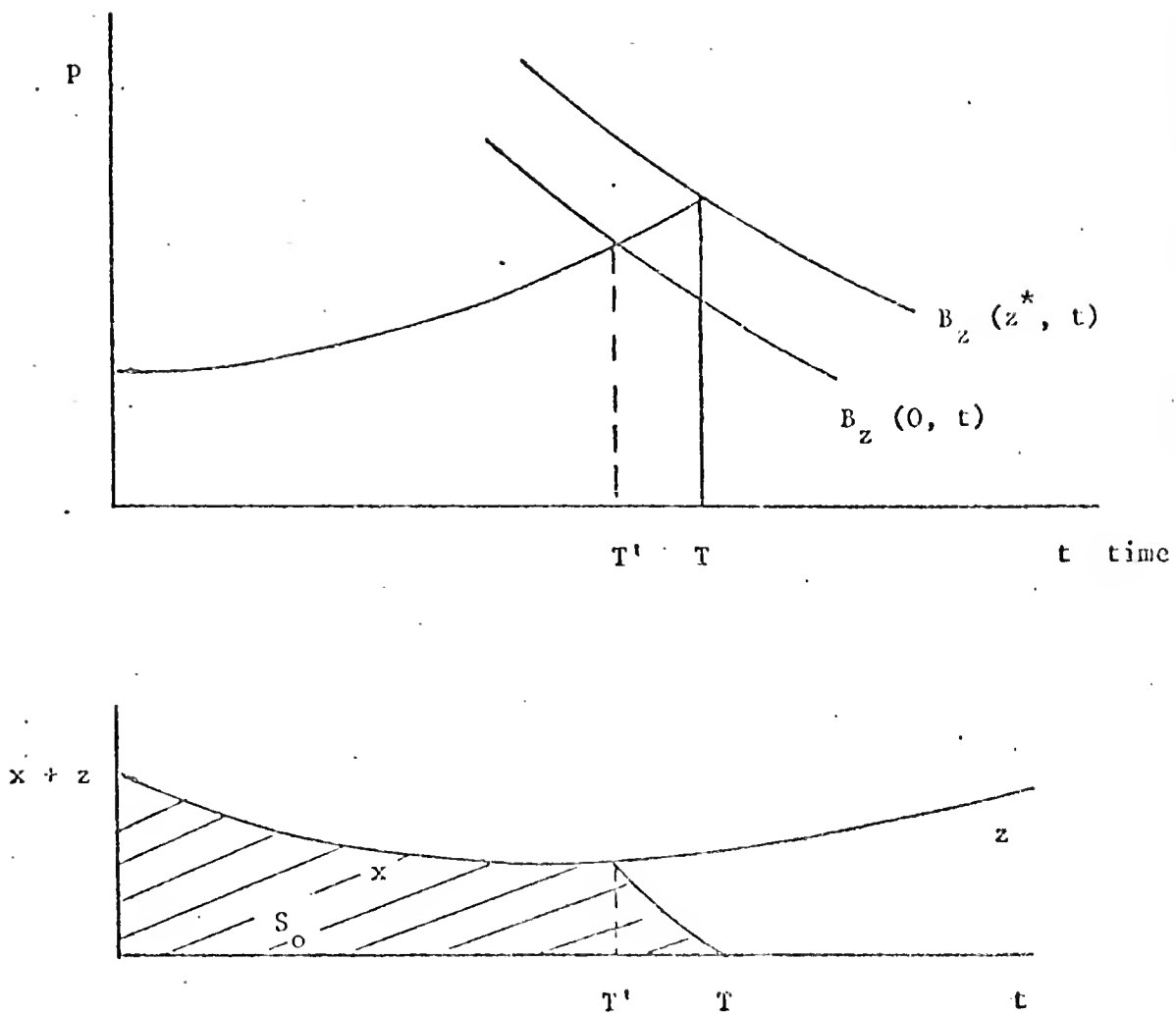


Figure 1 - Price and consumption paths

subject to the same conditions as before. As before

$$(6) \quad He^{rt} = (1 - \tau) (px - C) - \lambda x$$

$$p \leq c + \frac{\lambda}{1 - \tau}$$

$$\frac{d\lambda}{dt} = r\lambda + (1 - \tau) \frac{\partial C}{\partial S}$$

Since if  $\mu = \lambda / (1 - \tau)$  the equations are identical to (3), the same price equation results from eliminating  $\lambda$ , and, if the same boundary conditions are imposed, we have thus established

Proposition 1 : Any constant ad valorem tax on rent defined as revenue less current extraction costs imposed on a competitive exhaustible resource industry leaves extraction Pareto efficient.

An excise tax on the output of a normal competitive industry is distortionary unless supply is completely inelastic, just as monopoly control of such an industry is distortionary. However, we know that under some conditions (constant elasticity of demand, zero extraction costs) a monopolized exhaustible resource may be efficiently extracted and the monopolist may have no monopoly power. It turns out that for essentially similar reasons, though under a wider range of conditions, an excise tax on oil may be non-distortionary. Indeed, in special cases an excise tax will be identical to a pure rent tax, allowing all the surplus to be taxed away.

Let  $p$  be the consumer (after tax) price, and  $p_n$  be the net producer price, so that the excise tax  $\tau_e$  is  $p - p_n$ . Efficiency requires equation (4) to hold:

$$(4) \quad \frac{dp}{dt} = \frac{dc}{dt} + \frac{\partial C}{\partial S} + r(p - c), \quad 0 \leq t \leq T.$$

The producer price facing competitive resource holders must satisfy

$$(7) \quad \frac{dp_n}{dt} = \frac{dc}{dt} + \frac{\partial C}{\partial S} + r(p_n - c)$$

together with the complementary slackness conditions of equation (3), and the boundary conditions. Excise taxes will be efficient, if, subtracting equation (7) from (4)

$$(8) \quad \frac{d\tau_e}{dt} = r \tau_e$$

and

$$(9) \quad p_n - c \geq 0, \quad 0 \leq t \leq T.$$

The requirement that net rent remains positive is non-trivial, since it may severely limit the degree of excise taxation. In particular, if some oil is left in the ground because it is too expensive to extract, then the rent on this marginal oil will be zero, and there is no non-distortionary excise tax. Summarising, we have

Proposition 2 : An excise tax which rises at the rate of interest is non-distortionary provided that it does not drive producer rents below zero.

It is interesting to note that although both excise taxes and rent taxes are non-distortionary, the time profile of tax payments will differ if costs are stock dependent, for the rent tax payments per unit of oil  $T = \tau(p - C)$  will behave as

$$\frac{dT}{dt} = r T + \tau \frac{\partial C}{\partial S}$$

from equation (4), whilst the excise tax rises at the rate of interest. If costs are independent of stocks the two taxes are exactly equivalent.

The intuitive explanation of these results is that an excise tax rising at the rate of interest has constant present value, so that it acts as a lump sum tax provided it does not affect the amount of oil sold; that is, provided marginal oil is still worth extracting. It is equivalent to the static problem of an excise tax on an inelastically supplied commodity. If, in addition, marginal costs are constant and stock independent, the excise tax is exactly the same as the non-distortionary rent tax.

## 2. Optimum Tariffs for Competitively Supplied Imports

The previous section showed that an excise tax could be non-distortionary. Implicit in this demonstration, however, was the assumption that producers could rely on the price trajectory's satisfying (7) in the future. (7), the well-known arbitrage equation (see Dasgupta and Heal (1974)), must be satisfied if producers are to supply oil at each instant  $t$ . If, for example, they believed that at some future date producer prices were going to rise more quickly than (7) entails, producers would refuse to supply oil until that date.

As long as oil is bought and sold within national boundaries, (7) can be guaranteed by futures contracts or law. International trade in oil, on the other hand, is a prime example of a market where futures contracts are often impossible to enforce. This limitation has profound consequences. As we shall see, the impossibility of binding forward agreements may prevent a large country from benefitting from its market power (exercised through reduced imports, as with the conventional optimal tariff). Ironically, that



very market power may leave the country worse off than if it had no market power at all. Moreover, in such a framework, it is no longer possible to use optimal control theory to calculate the optimum tariff; a more sophisticated approach is required.

When agents' current decisions depend on their forecasts of the system's future evolution, decision-making is radically different from when agents act only on the basis of current and past observations. As Kydland and Prescott (1977) have observed in another context, optimal control theory is not then an appropriate tool for choosing a course of action. This point is clearly illustrated by the following example, in which we show why the tariff calculated by optimal control is incorrect.

To make the argument more transparent, assume that extraction costs are independent of stocks and flows and that every country has access to the backstop technology. The backstop can provide unlimited supplies of energy at a constant cost  $\bar{p}(t)$ . Our country derives dollar benefits  $U(x)$  from the consumption of  $x$  units of oil, and the demand by the rest of the world for oil of price  $p$  is  $y(p)$ . The problem is to choose a level of imports  $x$ , a production level  $z$  from the backstop technology, and a price trajectory  $p$  to maximize

$$(10a) \quad W = \int_0^{\infty} \{ U(x + z) - px - \bar{p}z \} e^{-rt} dt$$

subject to

$$(10b) \quad -\dot{S} = x + y(p)$$

$$(10c) \quad \dot{p} = \dot{c} + r(p - c) .$$

Equation (10c) is just the arbitrage equation. At some date  $T$  stocks of oil  $S(T)$  will be exhausted and  $p(T) = \bar{p}$ , with the rest of the world switching to the backstop technology. The Hamiltonian is

$$H = (U - px - \bar{p}z)e^{-rt} - \mu(x + y(p)) + \lambda(\dot{c} + r(p - c)).$$

Maximizing with respect to  $x, z$ :

$$\left. \begin{array}{l} U' \leq p + \mu e^{rt} \\ x \geq 0 \end{array} \right\} \text{complementarily}$$

$$\left. \begin{array}{l} U' \leq \bar{p} \\ z \geq 0 \end{array} \right\} \text{complementarily}$$

$$-\partial H / \partial S = \dot{\mu} = 0.$$

If  $q$  is the consumption price,  $q = U'(x)$ , then

$$(11) \quad \begin{aligned} p &= c + (p_0 - c_0)e^{rt} \\ q &= \text{Min} \{c + (q_0 - c_0)e^{rt}, \bar{p}\}, \end{aligned}$$

and the import tariff  $q - p$  rises at the rate of interest, just as did the efficient excise tax of the previous section. Indeed, given the cost assumptions, such a tax is equivalent to a rent tax.

But this will not do. If there were no other consumers, then our country could extract the entire surplus from the competitive producers. As it is, our country has to convince other consumers and producers that it will set an initial excise tax  $\mu$ , raise it at the rate of interest, and cease

importing at the point where the consumer price reaches the backstop price  $\bar{p}$ . But the import price will still be below  $\bar{p}$ , and it will then be rational for the country to change its tax plan and continue to import until  $p = \bar{p}$ . This policy is satisfactory as long as suppliers are naive enough not to anticipate it. But suppose they do foresee it. Naturally they will alter their own behaviour to take the impending tax change into account. This alteration in turn induces the large importer to change his tax policy, implying a further change in supply, and so on. It may not be clear that there is an equilibrium in which behaviour by each side is consistent with "rational expectations" about the actions of the other side. In fact, an equilibrium - though, a rather strange one - does exist, as we shall now demonstrate. To make the nature of the equilibrium transparent, we shall first strip our model of all inessentials.

### 3. A Simple Multi-period Model of Monopsony

Let us suppose that competitive oil producers must exhaust a stock of oil  $S$  in the first two periods before a cheap substitute becomes available in period 3. Extraction is costless, so that producers will plan to supply in both periods only if they expect the price at  $i$ ,  $p_i$ , to satisfy

$$p_1 = \beta p_2$$

where  $\beta$  is the world discount factor. A large oil importer,  $B$ , derives net utility

$$U'_B = a' \log x_1 + \beta \log x_2 - (p_1 x_1 + \beta p_2 x_2)$$

from consumption of oil  $x_i$  in period  $i$ . The rest of the world has demand for oil

$$(12) \quad y_1 = \frac{b'}{p_1}, \quad y_2 = \frac{1}{p_2}.$$

Demand by the 'rest of the world' can, to close the model, be thought of as demand by the producers themselves, and equation (12) as the solution to the following problem:

$$(13) \quad \text{Max } U_A = p_1 x_1 + \beta p_2 x_2 + b' \log y_1 + \beta \log y_2$$

$$(x_1, x_2, y_1, y_2)$$

subject to  $x_1 + x_2 + y_1 + y_2 = S$ .

For convenience, define new variables

$$a = a'/\beta, \quad b = b'/\beta, \quad U_B = U'/\beta$$

$$p = p_2, \quad \dots, \quad p_1 = \beta p$$

The phenomena we intend to study are essentially inter-temporal in nature and would not arise in a one-period world. The Arrow-Debreu model is essentially a one-period world, for with a complete set of futures and insurance markets agents can conclude all transactions in the first period and spend the rest of the time fulfilling contracts. To prevent our model from collapsing to this static one-period economy we assume that there are no futures markets, because of the absence of a global legal authority to enforce futures contracts. We shall, however, compare our equilibrium with the one which would have emerged with such contracts.

Initially we shall also assume that it is too costly to store oil above ground. We adopt this assumption/ <sup>because</sup> costless storage makes it possible to purchase second period oil in the first period, and might be thought to substitute for futures markets. Later (in sections 7 and 8) we shall

examine the extent to which storage can replace forward contracts.

As a reference point the competitive equilibrium  $(\bar{x}_1, \bar{x}_2, \bar{p})$  satisfies

$$(14) \quad \bar{p} = \frac{a + b + 2}{S}, \quad \bar{x}_1 = \frac{a}{\bar{p}}, \quad \bar{x}_2 = \frac{1}{\bar{p}}.$$

Notice that in this competitive framework, it makes no difference whether binding futures contracts exist or agents simply forecast future prices correctly. Agents' behaviour and the competitive outcome are the same regardless. We shall soon see that this coincidence exists precisely because no agent has market power and vanishes as soon as power is introduced.

One way to capture market power and the idea that buyer B is relatively large (Note: we shall throughout consider the case of a large buyer and small sellers. We could, of course, have just as easily considered the opposite symmetric case<sup>1</sup>) is to allow buyer B to behave as a Stackelberg leader. To behave as leader, in this case, means to choose the prices  $p_1$  and  $p_2$ , which other agents then accept as parametric. Since B chooses  $p_1$  and  $p_2$  at the same time - presumably in period 1 - we must specify whether  $p_2$  is in fact binding or whether, once period 2 arrives, agent B can change it. Let us first consider binding contracts. To be supplied in both periods, B must choose

$$\frac{p_1}{\beta} = p_2 = p$$

His maximisation problem is, therefore, to choose  $x_1, x_2$  and  $p$  to

$$(15) \quad \begin{aligned} \text{Max } U_B &= a \log x_1 + \log x_2 - p(x_1 + x_2) \\ \text{.. subject to } x_1 + x_2 &= S - \frac{1 + b}{p} = X(p). \end{aligned}$$

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1. We shall investigate the case of large buyers in future work.

Solving, we obtain

$$(16a) \quad \bar{x}_1 = \frac{a X(p)}{1+a}, \quad \bar{x}_2 = \frac{X(p)}{1+a}$$

which, when inserted into  $U_B$  give  $U_B(p)$ . Choosing  $p$  gives

$$(16b) \quad \bar{p} = \frac{1+b + \sqrt{(1+b)^2 + 4(1+a)(1+b)}}{2S}$$

Notice that we have considered  $B$  choosing import levels rather than import tariffs. Either formulation yields the same result, but the first is simpler. The implied tariff is simply  $q - p$ , where  $q$  is the demand price or dollar marginal utility. Thus

$$(17) \quad \tau_1 = \tau_2 = \frac{1+a}{X(p)} - p = \left( \frac{a+b+2-pS}{pS-(1+b)} \right) p$$

While  $(\bar{x}_1, \bar{x}_2, \bar{p})$  in equation (16) solves  $B$ 's two period maximization problem, that is not to say that, once period 2 arrives,  $B$  would not like to deviate from  $(\bar{x}_2, \bar{p})$ . In fact,  $B$  would like to deviate, in general. This is because the constraint  $\frac{p_1}{B} = p_2$  holds only for the original maximization problem of equation (15). Once period 1 has elapsed, the constraint is no longer binding, because time is irreversible and whatever transpired in the first period cannot be undone. Since we usually expect the solution to maximization problem to change after we drop a constraint it should not be surprising that  $B$  should want to break his contract.  $B$ 's optimal contract breach is given by the solution to the following problem:

Let  $S_1 = S - \bar{x}_1 - \frac{b}{p}$ , stocks at the start of period 2.

Choose  $p_2$  and  $x_2$  to maximize

$$(18) \quad \log x_2 - p_2 x_2 \quad \text{subject to} \quad x_2 + \frac{1}{p_2} = S_1$$

The solution is given by

$$(19) \quad p_2^* = \frac{g}{S_1}, \quad x_2^* = \frac{g-1}{p_2^*}, \quad \text{where } g \equiv \frac{1}{2} (1 + \sqrt{5}) \approx 1.618$$

It may be verified that  $p_2^* > \bar{p}_2$  for  $a > b$  and  $p_2^* < \bar{p}_2$  for  $a < b$ .

Only by coincidence-i.e., when  $a = b$ -will  $p_2^* = \bar{p}_2$ . Thus, it is unlikely that B will wish to fulfill his contract.

One interesting feature of the case  $p_2^* > \bar{p}_2$  (where B would like to break the contract by offering a higher second period price) is that both principal parties - B and the competitive suppliers - are, in fact, better off when B breaks the contract  $(\bar{p}_1, \bar{p}_2)$  than when contracts are binding. (To evaluate suppliers' welfare we assume that suppliers consume the "rest of the world" oil and are endowed with utility function (13).) Such a Pareto improvement, of course, is made possible by the fact that the binding contract configuration is not a competitive allocation. It has the peculiar implication that all parties to a trade agreement may agree to dispense with binding contracts in favor of an arrangement where binding contract prices prevail some of the time but are broken occasionally, and in a way unforeseeable to the suppliers, by the monopsonistic buyer.

We emphasized the phrase in the last sentence because the success of contract renegeing depends, of course, on the suppliers' not foreseeing it. If a supplier knew beforehand that, in the second period, B was going to raise the price offered from  $\bar{p}_2$  to  $p_2^*$ , he would defer selling any oil until the second period and if all suppliers knew, then B would fail to

be supplied in the first period at all. Notice too that the failure of contract reneging does not require perfect foresight (i.e., it is not necessary that the suppliers be able to calculate  $p_2^*$ ), only an awareness that the monopsonist has an incentive to raise the second period price. If suppliers have rational expectations in this rather weak sense, agent B faces a dilemma. Even if he is an "agent of good faith" and promises to stick to the binding contract prices, he will not, in the absence of enforceable contracts, be able to be supplied in the first period (if  $p_2^* > \bar{p}_2$ ) or the second period (if  $p_2^* < \bar{p}_2$ ). His promises will simply not be credible. This dilemma is reminiscent of the problem of commitment, in the game theory literature (see, for example, Schelling (1963)). In many games, a player would be better off if somehow he could commit himself beforehand to pursue a certain strategy. Commitment would force other players to optimize with respect to his own strategic choice. Inability to self-commit on the other hand might induce the other players to choose strategies which are less favorable for him.

Let us call the situation in which B acts according to the bonding contract equilibrium in the first period and then reneges in the second period, the reneged contract equilibrium. As we have said, such an equilibrium relies on suppliers' having myopic or static price expectations in the sense that they believe that the present value of tomorrow's price is the same as today's. If, however, B can rely on the suppliers' having myopic expectations, he may do even better than the reneged equilibrium by setting  $p_1$  other than at the binding contract level. As before, B's second period problem is given by (18), and the solution is given by ..

$x_2 = \frac{\varepsilon_1 (g-1)}{g}$  ,  $p_2 = \frac{g}{\varepsilon_1}$  . Supply in the first period is as in equation (16a) with  $p_1$  substituted for  $p$  ( $p_1$  need equal  $p_2$  because suppliers' expectations are myopic):



$$x_1 = \frac{a}{1+a} (S - \beta(1+b)/p_1) = S \frac{b\beta}{p_1} - s_1,$$

whence

$$s_1 = \frac{S + \frac{a-b}{p_1}}{1+a}.$$

B's overall problem is to

$$\text{Max}_{p_1} \quad a \log S - \frac{(1+b)\beta}{p_1} + \log S + \frac{(a-b)\beta}{p_1} - \frac{ap_1 S}{(1+a)\beta}$$

terms

where constant/ have been ignored. Setting  $S = 1$  for simplicity, the solution is given by the cubic

$$ap_1^3 - a(1+2b-a)\tilde{p}_1^2 - (b(1+a)^2 + a(a-b)(1+b))\tilde{p}_1 + (1+b)(b-a)(1+a)^2 = 0,$$

$$\text{where } \tilde{p}_1 = p_1/\beta.$$

In the numerical example calculated at the end of the next section, B does considerably better by exploiting myopic price expectations than he does by starting along the implicit contract path only to subsequently deviate from it.

#### 4. Equilibrium with Rational Expectations

A more complex question is how agent B ought to behave when suppliers have intelligent predictive abilities rather than myopic expectations. The answer is that, if he is to be supplied in both periods - which he certainly will require, given the logarithmic terms in his utility function - he must choose  $p_1$  and  $p_2$  not only, subject to the constraint that  $p_1 = \beta p_2$  but also so that, once the second period arrives, he has no incentive to alter  $p_2$ . Only then can he convince suppliers that  $p_2$  will actually prevail. To solve B's problem we must work back from the last period. Suppose, then, that  $p_1$  and  $x_1$  prevailed in period 1, leaving a stock of oil

$$S_1 = S - x_1 - \frac{b\beta}{p_1}.$$

Agent B's second period maximisation problem is:

$$(20) \quad \text{Max } \log x_2 - p x_2 \quad \text{subject to } x_2 = S_1 - \frac{1}{p}$$

where B chooses  $x_2$  (or, equivalently,  $p$ , which is simpler).

This yields

$$p S_1 = g (= \frac{1}{2} (1 + \sqrt{5}))$$

or

$$(21) \quad x_2 = \frac{g - 1}{p}$$

It is interesting to note that here, as in the reneged contract of equation (19), the tariff required to sustain this import level is

$$\frac{\tau_2}{p} = -\frac{x_2}{p} - 1 = g - 1 = .618$$

In addition,  $p_1 = \beta p$ , so

$$(21b) \quad x_1 = S - \frac{G}{p}, \quad G \equiv g + b.$$

Substituting these values of  $x_i$  in the full maximization problem:

$$(22) \quad \max_p \quad a \log \left( S - \frac{G}{p} \right) + \log \left( \frac{g-1}{p} \right) - p \left( S - \frac{1+b}{p} \right)$$

whence

$$(23) \quad p = \frac{G-1 + \sqrt{(G+1)^2 + 4aG}}{2S}.$$

Since B has this equilibrium forced upon him, it is obvious that it makes him worse off than the binding contract equilibrium, which he would choose if he could. What is more surprising is that the Rational Expectation Equilibrium (REE) can make B worse off than in the competitive equilibrium, as the example below shows. In such cases B's market power is his undoing, and he would be better off without it. His problem is that there is no obvious way he can convince the rest of the world that he is renouncing this power (short of granting full fiscal autonomy to its States). It is B's potential ability to change future prices that induces expectations and responses in the other agents which are so unfavourable to him.

## 5. A numerical example

Consider the above model with parameter values

$$a = 0.2, \quad b = 5, \quad S = 1$$

	Competitive (1)	Binding Contract (2)	Reneged Contract (3)	Myopic forecasts (4)	REE (5)
$p_1$	7.2	7.025	7.025	9.077	6.788
$p_2$	7.2	7.025	6.129	4.121	6.788
$\tau_1/p_1$	0	0.171	0.171	-0.390	0.177
$\tau_2/p_2$	0	0.171	0.618	0.062	0.618
$x_1$	0.028	0.024	0.024	0.057	0.025
$x_2$	0.139	0.122	0.100	0.229	0.091
$U_A + 4$	1.403	1.375	1.275	1.057	1.344
$U_B + 5$	1.109	1.125	1.173	1.495	1.079

Note :  $U_A$ ,  $U_B$  have had constants added to produce positive numbers.

Observe that  $U_B^4 > U_B^3 > U_B^2 > U_B^1 > U_B^5$ , where superscripts refer to equilibria. B's market power makes him worse off in the REE than if he were a competitive importer and could divide himself up into small importing units, thereby divesting himself of the ability to set prices. Aggregating suppliers with other consumers as in equation (13) we notice that

$$U_A^1 > U_A^2 > U_A^5 > U_A^3 > U_A^4$$

and suppliers would also prefer the binding contract to either the REE or reneged contract equilibrium. Both/ sides of the market would be interested in finding a means of credibly enforcing contracts. If we distinguish between producers and the/ competitive consumers, this harmony of interest vanishes - the consumers prefer 5 to 3 to 2 to 1 to 4.

In this example the monopsonist is disadvantaged because he places a relatively high premium on second period imports, and has relatively greater

monopsony power then. (In competitive equilibrium B counts for only 4% of the first period demand, but 50% of the second period's.) His incentive to renege on the initial tariff plan is thus great, and can only be eliminated by reducing second period supplies to the point where his great need for consumption offsets his monopsony power. This is done by producers, who, fearing a low second period price, sell more in the first period and so drive the price below the binding contract level. The example is rather extreme, but bears an unpleasant similarity to the U.S. position, itself derived from past profligacy in consuming domestic oil stocks. The boundary values for  $a$ ,  $b$ , such that the REE is no worse than the competitive equilibrium roughly satisfy  $b^* = 2 + 4.6 a^*$ , for  $0 < a < 1$ . For  $b < b^*$  or  $a > a^*$  the monopsonist is advantaged by his market power.

## 6. Quotas

Notice that in the preceding example  $p_2^* < \bar{p}_2$ ; that is, B would like to renege on the second period price so as to lower this price. In the REE this incentive has been removed. But suppose B can impose import quotas in the first period. Suppliers, forecasting a fall in price in period 2, try to sell everything in the first period, but fail, as B refuses to import more than  $\bar{x}_1$ , and are forced to sell at the lower second period price. Does this mean that B can take actions in the first period to force suppliers to accept the reneged contract equilibrium (or some alternative even more favourable to B)? Does B benefit from imposing quantity constraints which prevent the market from clearing? Perhaps surprisingly, the answer is no.

If the other consumers are distinct from the suppliers, then B will be unsuccessful in preventing  $p_1$  falling to  $p_2$ , since suppliers will off-load oil onto the remaining consumers until  $p_1 = p_2$ . Suppose therefore that

that

the suppliers themselves are the remaining consumers and/ if they fail to sell oil to B they are forced to either consume it themselves or retain it for the second period. Then, the opportunity cost to the suppliers of consuming oil is not the high price  $p_1$ , but the lower price  $p_2$ . This means that the only question affected by  $p_1$  is how much B pays for his period 1 purchases. Thus B might as well drive  $p_1$  down as low as is consistent with being able to purchase what he wants - that is, down to  $p_2$ . A similar argument holds when  $p_2^* > \bar{p}_2$ . Thus it is impossible to sustain different prices in the two periods with rational expectations, and so quotas lead to the same equilibrium as before.

## 7. storage

The ability to store oil costlessly gives the buyer another control variable and, therefore, the potential, in principle, to improve upon equilibrium without storage. Actually, two equilibrium solutions - the competitive and binding contract equilibria - remain unaltered with storage; in both cases, all transactions are effectively concluded in the first period, and agents lack either the power or the right to renegotiate in the second period. Storage, therefore, makes no difference. Matters are different, however, if B is not constrained by a contract in period 2. If he knows that the rest of the world is myopic, B's problem is to choose price  $p_i$ , purchases  $X_i$ , and consumption  $x_i$  in period  $i$  to

$$\begin{aligned} \text{Max } a \log x_1 + \log x_2 - ((p_1/\beta)X_1 + p_2X_2) \\ \text{subject to } X_1 \leq S - \frac{(1+b)\beta}{p_1}, \quad X_2 \leq S - X_1 - \frac{b}{p_1} - \frac{1}{p_2} \end{aligned}$$

(24)

$$x_1 \leq X_1, \quad x_1 + x_2 = X = X_1 + X_2, \quad S - X_1 - \frac{b}{p_1} - \frac{1}{p_2} \geq 0$$

If the solution to (24) entails  $x_1 = X_1$ , then it is identical to the myopic equilibrium of section 3. If  $X_1 > x_1$ , then replacing  $p_2$  by  $p$ , B's optimal

consumption, is, as in equation (16a), to set

$$x_1 = \frac{a}{1+a} X, \quad x_2 = \frac{X}{1+a}, \quad X = 1 - \frac{bp}{p_1} - \frac{1}{p}.$$

whereupon the problem of (24) simplifies to

$$(25) \quad \text{Max}_{p, p_1} (1+a) \log X - (p_1 + \frac{p}{p_1}).$$

The solution requires the prices  $p, p_1$  to satisfy

$$(26a) \quad p = \frac{p_1}{2B} \{ \sqrt{4bp_1+1} - 1 \}$$

and

$$(26b) \quad p = \frac{p_1}{2(p_1-b)} \{ 1 + \sqrt{1+4(1+a)(p_1-b)} \}.$$

Clearly B can do no worse with storage than without since he can always take  $x_1 = X_1$ ; that is, he can store nothing. It is of some interest to see that there are instances where storage makes him strictly better off. Consider the symmetric case in which  $a = b = 1$ . Without storage, the binding contract, reneged contract, myopic, and RE equilibria are identical because B has no incentive to change the second period price. If storage is to make a difference for the myopic equilibrium, then the solution to (24) when  $a = b = 1$  must entail  $X_1 > x_1$ , in which case  $p$  and  $p_1$  can be calculated from equations (26). That storage does indeed make a difference is demonstrated by the following table:

## Myopic Equilibrium

$$a = b = 1 \quad s = 1$$

	<u>Without Storage</u>	<u>with storage</u>
$p_1$	3.236	2.968
$p_2$	3.236	3.840
$x$	0.382	0.403
$t_1/p_1$	0.62	0.67
$t_2/p_2$	0.62	0.29
$U_B + 5$	0.453	0.533
$U_A + 4$	2.887	2.828

8. Rational Expectations with Storage

We suggested earlier that the ability to store oil might serve as a substitute for finding futures contracts by enabling B to buy all his oil in the first period. When suppliers have rational expectations, however, B may not be able to collapse the model to a single period in this way. This is because once the second period arrives, he may have an incentive to reopen the market and buy more oil. This incentive will be perceived by foresightful suppliers, who will consequently thwart B's first period plans. A rational expectations equilibrium with storage, therefore, is not entirely straightforward.

Even the matter of where oil is stored is potentially of some consequence under rational expectations. Suppliers must take into account storage by B to determine their current supply. Presumably before he buys his first period oil, B announces how much he intends to store. If suppliers themselves (or some trustworthy third party) perform the storage they can be sure that, once B has bought his first period oil, he cannot renege on this announcement. Therefore, under these arrangements, B can commit himself



to store any quantity he pleases. On the other hand, if B stores oil within his own boundaries, where he is not subject to surveillance, nothing can prevent him from reneging. Since we assumed rational expectations, B is constrained to announce a level of storage from which he would not want to deviate after buying his first period oil. By analogy with Section 4, one would conjecture that this additional constraint would make B worse off.

Because it is simpler, we first examine the case where B can commit himself in advance to storing a (known) amount  $x$ . Again, letting purchases be  $X_1$  and consumption,  $x_1$ , his second period problem is

$$(25) \quad \max \log (X_2 + z) - pX_2 \quad \text{subject to } X_2 = S_1 - \frac{1}{p},$$

which yields

$$(26a) \quad x_2 = \frac{1}{p^2 s_1} = X_2 + z = S_1 - \frac{1}{p} + z$$

$$(26b) \quad S_1 = \frac{1 - zp + \sqrt{(1 - zp)^2 + 4}}{2p}$$

This solution is substituted into the first period problem

$$(27) \quad \max_{z, p} a \log x_1 + \log x_2 - p (S - (1 + b)/p)$$

Such that  $x_1 = S - \frac{b}{p} - (s_1 + z)$ ,  $x_2 = S_1 + z - \frac{1}{p}$ ,  $z \geq 0$

Differentiating (27) with respect to  $z$  gives

$$(28) \quad \left( \frac{x_1 - ax_2}{x_1 x_2} \right) \left( 1 + \frac{as_1}{az} \right) \leq 0 \quad \left. \vphantom{\frac{x_1 - ax_2}{x_1 x_2}} \right\} \text{complementarily}$$

$$z \geq 0$$

suppose that  $z > 0$  at the optimum. Because

$$1 + \frac{\partial s_1}{\partial z} = \frac{1}{2} - \frac{1}{2} \frac{1 - zp}{((1-zp)^2 + 4)} > 0 \quad \text{we can conclude from (28) that}$$

$$(29) \quad x_1 = ax_2 = \frac{a}{1+a} (S - (1+b)/p)$$

Differentiating (28) with respect to  $p$  yields

$$(30) \quad \frac{x_1 - ax_2}{x_1 x_2} \frac{\partial s_1}{\partial p} + \frac{1}{p^2} (1/x^2 + ab/x) - S = 0.$$

Substituting from (29) gives

$$(31) \quad p = \frac{1 + b + \sqrt{(1+b)^2 + 4(1+b)(1+a)}}{2S}$$

From (27),  $x_1 = S - b/p - (s_1 + z)$ . Substituting, using (29), (26b), and (31), in this last equation, we deduce that  $z > 0$  if and only if  $a < b$ . But equations (29) and (31) are equivalent to (16a) and (16b), the equations for the binding contract equilibrium. We conclude, therefore, that if  $a < b$ , the RE equilibrium with storage is equivalent (same price, same consumption) to the binding contract equilibrium. For  $a \geq b$ ,  $z = 0$  in equilibrium, and so rational expectations equilibrium with storage is the same as RE equilibrium without storage. Summarizing, if B places relatively greater weight on the future ( $b > a$ ), then the possibility of storage makes a difference, and, in fact, storage serves as a perfect substitute for binding futures contracts and an agreement not to reopen spot markets. If, however, B cares relatively more about the present ( $a > b$ ), no storage will occur in equilibrium, and so equilibria with and without storage coincide.

From the concluding paragraph of section 5, we know that the no-storage REE can leave the monopsonist worse-off than the competitive equilibrium only when  $a < b$ . Therefore, the paradox of disadvantageous monopsony disappears when storage becomes a possibility.

The obvious question to ask is whether this dramatic improvement of B's fortunes in the case  $b > a$  depends on his publicly committing hostages, in the form of stored oil, to fortune. What would happen if storage took place unobserved on his own territory? In such a case the rest of the world observes first period purchase  $X_1$ , but not  $z$ , so that B chooses  $z$  and  $X_2$  (or  $p$ ) to

$$\text{Max } a \log (\bar{X}_1 - z) + \log (z + \bar{S}_1 - \frac{1}{p}) - p(\bar{S}_1 - \frac{1}{p})$$

Choosing  $z$  yields equation (29) again, choosing  $p$  gives (26a). This solution is fed into the first period problem of equation (27), and, again <sup>Thus</sup> the binding contract equilibrium results. / it makes no difference whether B's consumption and storage decisions are observable with rational expectations, because agents can predict or deduce them.

## 9. Discussion

If the government has tax jurisdiction over oil suppliers and consumers the optimal excise tax rises at the rate of interest. If the government can enter binding futures contracts with foreign suppliers, then the optimal tariff will look exactly the same, as it will if oil can be costlessly stored, the rest of the world has rational expectations, and  $B$  wishes he could lower the final period price below the binding contract level ( $b \geq a$ ). However, if storage is costly, binding futures contracts do not exist, and complete tax jurisdiction is impossible, <sup>then</sup> the optimal tariff changes dramatically. If suppliers are myopic, then the importer will continually revise his tariff, and depart from the apparently optimal plan. If suppliers are sophisticated enough to appreciate this temptation, or if they learn from experience that the import tariff is continuously revised, then they will change their behaviour considerably, and thus greatly alter the optimal import tariff. Their response can, in fact, make the monopsonist importer worse off than if he were to behave competitively. This paradoxical result was demonstrated in a very simple two-period model and we should ask whether the model was very special, or whether the results are robust. (In defence of the model, it should be said that although it is the simplest model capable of exhibiting the paradox, it is

surprisingly rich in the variety of equilibrium concepts it can display, and we think gives remarkably clear insights into the issues).

One obvious criticism of the model is that the supply response is perfectly elastic, so that producers sell only in the periods of highest expected price, and not at all in any period of lower expected price. If marginal extraction costs are increasing this will no longer be true, so that a variety of intertemporal price paths will be consistent with profit maximizing behaviour. The appendix shows, however, that this criticism does not weaken the results (though it greatly increases the complexity of the problem.)

The paradoxical result in which the monopsonist is harmed by his market power arises only when he has an incentive to lower the final period price. This induces the suppliers to drive down the earlier prices, which results in other consumers buying too much. If he wishes to raise the final period price he cannot be made worse than acting as a competitive importer, for if he had imported the competitive level of imports earlier on, suppliers would have predicted that he would drive down the final period price. Somewhere between the reneged contract plan and the competitive plan is one which leaves no incentive to deviate (i.e. the REE), and it will yield an intermediate level of utility.

This argument implies that if the backstop is available (at some cost) before oil is exhausted, and if the price of oil is set by this backstop, (as in the original optimal control problem) then the monopsonist does derive at least some benefit from his market power, and there is no paradox. This follows because the binding contract plan requires him to switch to the backstop before other consumers, and makes it attractive for him to re-enter the market and increase prices later. Thus a paradox is more likely if a dramatic breakthrough in oil substitutes is anticipated, and less likely if

there is a smooth transition from oil to the backstop. On the other hand, even if the paradox does not apply, it remains true that the optimum tariff with rational suppliers must be calculated recursively, as in the simpler model, and does not satisfy the appealing rule (of increasing at the rate of interest) of the optimal excise tax. It is also true that the monopsonist will typically be strictly worse off than with binding constraints.

What happens if there are more than two periods? It should be clear from the examples that it is difficult to characterise the general form of the REE. The problem is that market power differs in each period, and in general there is no simple relationship between consumption and the choice variable, price. There is, however, one special case in which the REE is readily calculable - when it coincides with the binding contract equilibrium.

#### 10. The consistency of simple tax rules and repeated games

If, in the simple model,  $a = b$ , B has no incentive to change  $p_2$ , and the three equilibria (binding contract, reneged contract and RE) coincide. The same is true if  $y_1 = cb/p_1$ ,  $y_2 = c/p_2$  and  $a = b$ . This naturally prompts the question when "When will the monopsonist have no incentive to deviate from his initial plan? "When, in short, does the REE collapse into the binding contract equilibrium? The following model shows how stringent are the conditions for this to happen, and given some insight into the general requirements.

Consider a model with zero extraction costs, with no backstop available before  $T$ , and initial stocks small enough to guarantee exhaustion by  $T$ . Let the price be  $p$ , the ad valorem tariff rate be  $\tau$  (constant on the binding contract path) and let both B and the rest of the world exhibit constant elastic demand:

$$y(p) = p^{-\epsilon}$$

$$V(p) = \frac{ap^{1-\alpha}}{\alpha-1} \quad \alpha > 1$$

where  $V(p)$  is B's indirect function. Post-tariff net present discounted welfare will be

$$W = \frac{ap_0^{1-\alpha} (1+\tau)^{-\alpha} (1+\alpha\tau) \phi(\alpha r)}{\alpha-1}$$

where  $p_0$  is the initial price,  $\phi(m) \equiv \frac{1}{m} (1 - e^{-mT})$  is a discount factor, and  $r$  is the rate of interest and the rate of increase of  $p$ . If  $X, Y$  are the total consumption of B and the rest of the world,

$$X = ap_0^{-\alpha} (1+\tau)^{-\alpha} \phi(\alpha r)$$

$$Y = p_0^{-\epsilon} \phi(\epsilon r)$$

$$X + Y = S, \text{ initial stocks of oil.}$$

The optimal choice of  $\tau$  satisfies

$$\tau = \frac{X}{\epsilon Y} = \frac{ap_0^{\epsilon-\alpha} (1+\tau)^{-\alpha} \phi(\alpha r)}{\phi(\epsilon r)}$$

In general  $\tau$  will depend upon  $T$ , and, through  $p_0$ , on the initial stock  $S$ . As time elapses these will both change and B will wish to change  $\tau$ . However, in the special case in which  $\epsilon = \alpha$ ,

$$\epsilon \tau = a(1+\tau)^{-\epsilon}$$

solves for  $\tau$ , which is independent of time (and stocks). Only in this very special case will the REE coincide with the binding contract equilibrium solved here. If extraction costs are not constant, or if both parties do

not have inelastic demand curves, then, as time passes, the ratio

$$\frac{x(p(1+\tau))}{y(p)}$$

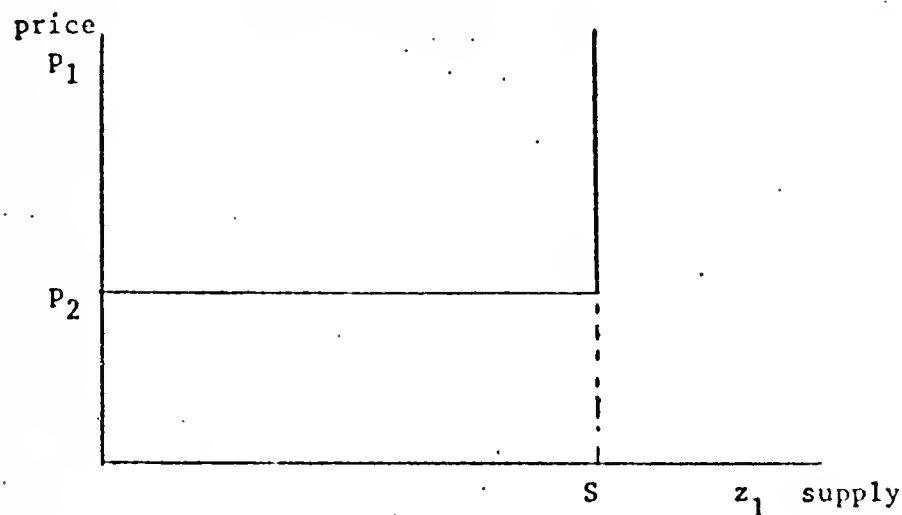
will change, changing the optimum tariff. If  $p_T$  is fixed (by a backstop) then the original argument shows that  $\tau$  changes over time.

The other case in which B has no incentive to alter the tariff again occurs when the future continues to appear the same (in the relevant sense) with the passage of time. If the remaining oil stock is unknown, then Gilbert (1978) has shown that if the probability distribution for the stock remaining is stationary, the optimal extraction rate will be constant (if costs and demands are also stationary). In such a world the optimum tariff would also remain constant, for the future will always look like the present. (To some extent this finesses the problem by making oil quasi-inexhaustible and hence like a conventional produced good). Another way the future can be rendered stationary is to consider a repeated version of a one-shot game. If our multi-period oil game could be played over and over again, we would find that the binding contract equilibrium emerges as a possible REE, as long as players' discount rates are not too high. This follows for much the same reasons that cooperative behaviour becomes viable in an indefinite repetition of the Prisoner's Dilemma. The market for oil, therefore, is an especially good vehicle for demonstrating our paradoxical results. An exhaustible resource, by its very nature, lends itself to a non-repeatable theoretical formulation.



# Appendix - The effects of variable extraction costs

It might be thought that the paradox derives from the special assumption that the marginal costs of extraction for the producers are zero in each period, so that supply curve is a perfect step function, with



equilibrium on the horizontal section. This is not so, though it becomes more difficult to construct examples when the supply curve is a function of the intertemporal pattern of prices. As an illustration, suppose the present discounted marginal costs of supplying  $z_i$  in period  $i$  are  $mz_i$ . (Again, it is easier to redefine units so that the rate of interest is zero.) The suppliers, expecting prices  $p_i$ , maximize profits

$$\sum (p_i z_i - \frac{m}{2} z_i^2) \quad \text{st} \quad \sum z_i \leq S$$

which yields supply curves

$$z_1 = \frac{1}{2} (S + \frac{1}{m} (p_1 - p_2))$$

$$z_2 = \frac{1}{2} (S - \frac{1}{m} (p_1 - p_2))$$

The effect of this supply response is to change the equilibrium intertemporal price structure from  $p_1 = p_2 = p$  to a more complex relationship  $p_1 = f(p_2) = f(p)$ , whose form will depend on the market structure (i.e. on the equilibrium concept). Thus, in the case of the competitive equilibrium

$$z_2 = \frac{2}{p_2} = \frac{1}{2} (S - \frac{1}{m} (p_1 - p_2))$$

or 
$$p_1 = p + m(S - \frac{4}{p}) = f_c(p)$$

The prices can be found as before from

$$\frac{a+b}{f(p)} + \frac{2}{p} = S$$

The solution is continuous in  $m$  (the slope of the supply curve) and as  $m$  tends to zero, so the new equilibrium converges to the equilibrium in the original problem. In the Stackelberg equilibrium we again have

$$x_2 = \frac{g-1}{p_2}, \quad y_2 = \frac{1}{p_2}$$

$$z_2 = \frac{g}{p_2} = \frac{1}{2} (S - \frac{1}{m} (p_1 - p_2))$$

or 
$$p_1 = p + m(S - \frac{3.236}{p}) = f_s(p)$$

Notice the slight difference in functional form between the competitive and Stackelberg price patterns. The solution is found by noting that  $x_i$  are (more complex) functions of  $p$ , hence so is utility, and it can be maximized for a suitable choice of  $p$ . Again, the functions are continuous in  $m$ , from which it follows that there is a sufficiently small value of  $m$  which yields the paradox again.

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